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Current relaxation in disordered one- and two-dimensional systems

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Abstract. The current relaxation of disordered one- and two-dimensional systems is treated on the basis of a self-consistent approach for the diffusion coefficient. At the predicted delocalization field strength a long-time power law decay of the current is observed according to $j(t) \simeq t^{-1/2}$ if inelastic scattering processes are neglected. This slow current relaxation should be measurable in one-dimensional systems. The experimental verification of the results for a two-dimensional electron gas is complicated on account of the fact that both the delocalization field strength and the characteristic decay time depend exponentially on the disorder parameter. The measurement of the slow 2D current relaxation requires very low carrier densities and extremely low temperatures.

1. Introduction

The effect of a DC electric field on the Anderson localization of electrons in disordered solids has been quite a controversial issue for many years. It had been realized that carrier heating due to the electric field plays an important role in the transport properties of semiconductor microstructures. However, the direct electric field effects on the quantum interference observed in disordered two-dimensional systems had been accounted for differently. According to Altshuler *et al* [1] in contrast to a high-frequency field a constant electric field does not directly alter the quantum corrections to the conductivity, since the underlying time reversal symmetry is not violated. Qualitatively the same results were obtained by Lei and Cai [2] and Hershfield and Ambegaoker [3] who included coherent backscattering processes in their balance equation and generalized Boltzmann equation approach, respectively. An influence of the electric field on the collision integral was excluded. Non-linear electric field effects on the resistivity were treated in [2].

On the other hand Kirkpatrick [4] extended the self-consistent theory of Vollhardt and Wölfle [5] to treat the influence of a finite electric field and obtained the result that all two-dimensional states are delocalized even in very small electric fields. A direct electric field effect on the Cooperon was also predicted by Tsuzuki [6].

For a one-dimensional disordered solid there are exact and rigorous results for the field dependent electron localization [7, 8]. At small but finite electric fields the theories predict a power law localization rather than an exponential dependence typical for electrons at zero field strength. At some critical value of the electric field there is a mobility edge above which the electrons are delocalized. These 1D results were qualitatively reproduced by Kirkpatrick's self-consistent approach too [4].

Recently we extended the self-consistent theory of localization worked out by Vollhardt and Wölfle [5] to treat the influence of small electric and magnetic fields [9]. In the present paper we apply this approach to investigate the peculiarities of the current relaxation of disordered one- and two-dimensional systems.

2. Quantum diffusion in an electric field

The starting point of our phenomenological approach in [9] was the following self-consistent equation for the diffusion coefficient D:

$$D = D_0 \Bigg/ \left(1 + \frac{1}{\hbar\pi N_{\mathrm{F}}} \sum_q C(q)\right). \tag{1}$$

Here $D_0 = v_{\mathrm{F}}^2\tau/d$ is the bare diffusion coefficient in a d-dimensional lattice obtained from the Boltzmann transport equation (v_{F} is the Fermi velocity, N_{F} the density of states at the Fermi surface† and τ the elastic scattering time). The Cooper propagator $C(q)$, from which the renormalized autocorrelation function is obtained by a summation over q, was calculated from the Laplace transformed quantum diffusion equation

$$[s + q^2 D + \mathrm{i}qE\mu]C(q) = 1 \tag{2}$$

In the thermal equilibrium ($E = 0$) the parameter s of the Laplace transformation can be identified by the complex frequency $-\mathrm{i}\omega$. As in [9] here we consider the restricted electric field region where we can assume that the Einstein relation between the renormalized mobility μ and the diffusion coefficient is still valid:

$$\mu = eDN_{\mathrm{F}}/N. \tag{3}$$

N is the electron concentration, which is related to the Fermi energy ε_{F} by $N/N_{\mathrm{F}} = 2\varepsilon_{\mathrm{F}}/d$. For a two-dimensional lattice ($d = 2$) from (1)–(3) a delocalization field strength E_0 was predicted, which depends exponentially on the disorder parameter $k_{\mathrm{F}}\lambda$ ($\lambda^2 = 2D_0\tau$ is the elastic scattering length):

$$E_0 = \frac{2\kappa\varepsilon_{\mathrm{F}}}{e} \exp\left(-\frac{\pi}{2}k_{\mathrm{F}}\lambda\right). \tag{4}$$

The upper momentum cut-off of the integral in (1) denoted by κ has to be determined by a physical argumentation given below. Below this delocalization edge the current vanishes and above E_0 the current depends logarithmically on the electric field according to

$$j(E) = e^2 N_{\mathrm{F}} E \left[D_0 - \frac{1}{2\pi^2\hbar N_{\mathrm{F}}} \ln\left(\frac{2\kappa\varepsilon_{\mathrm{F}}}{eE}\right)\right]. \tag{5}$$

For a one-dimensional lattice we reproduced the exact value of the delocalization field strength obtained by Prigodin [8]. These results were obtained in the limit $s \to 0$ where we completely neglected inelastic scattering processes.

Our phenomenological approach was supported by physical arguments, which follow from a quantum diffusion picture [9]. According to this reasoning the static electric field gives rise to an upper cut-off time for the return probability because the diffusion

† In [9] the definition of N_{F} includes a factor of two in addition.

volume moves with the constant velocity μE. Up to a numerical factor the predicted field dependence of the diffusion coefficient agrees with (5).

Inelastic scattering can phenomenologically be treated by introducing an inelastic scattering time τ_ε ($s \to s + 1/\tau_\varepsilon$, cf, e.g., [2]). Due to inelastic scattering the current does not vanish below the field strength E_0, rather with decreasing τ_ε the delocalization edge is increasingly washed out. In this case the field dependence is obtained from the numerical solution of the self-consistent equation for the diffusion coefficient as discussed in [9].

Here we focus our attention on the time dependence. If there is no electric field (thermal equilibrium) s can be identified by the complex frequency $-\mathrm{i}\omega$ and the frequency dependent diffusion coefficient $D(\omega)$ may be calculated and analysed from (1) and (3). However, if there is a non-linear field dependence of the current a frequency representation of the diffusion coefficient is not appropriate because one does not expect an oscillating time dependence. Therefore, the time dependent current is considered, which results if initially at $t = 0$ a constant electric field is switched on (i.e. $\boldsymbol{E}(t) = \boldsymbol{E}\Theta(t)$, where $\Theta(t)$ is the step function). Using the Einstein relation (3) the time dependence of the current is obtained from the inverse Laplace transformation

$$j(t) = e^2 N_\mathrm{F} \boldsymbol{E} \frac{1}{2\pi\mathrm{i}} \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}s}{s} \mathrm{e}^{st} D(s, \boldsymbol{E}) \tag{6}$$

where $D(s, \boldsymbol{E})$ is the solution of the self-consistent equations (1) and (2). In the following sections we derive explicit expressions for the time dependent current for disordered one- and two-dimensional systems.

3. Current relaxation in two-dimensional systems

Because the localization of two-dimensional electrons in a constant electric field is quite a controversial subject we start the discussion with this case. The q integral in (1) is easily calculated and we obtain from (6) the following equation for the time-dependent current:

$$j(t) = \frac{e^2 E}{4\pi^2 \hbar} f(t) \qquad f(t) = \frac{1}{2\pi\mathrm{i}} \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}\sigma}{\sigma} \exp(\sigma t/T) f(\sigma) \tag{7}$$

where we introduced the dimensionless variable $\sigma = sT$ and the characteristic time

$$T = \frac{4\pi^2 \hbar N_\mathrm{F}}{\kappa^2} \exp\left(\frac{\pi}{2} k_\mathrm{F} \lambda\right). \tag{8}$$

In (7) $f(\sigma)$ is the dimensionless current $f(\sigma) = 4\pi^2 \hbar N_\mathrm{F} D(\sigma)$, which, according to (1)–(3), obeys the self-consistent equation

$$f = \ln\left[\frac{\sigma + T/\tau_\varepsilon}{f} + \varepsilon^2\right] \tag{9}$$

and which depends on the electric field via $\varepsilon = E/E_0$. To proceed further we transform (7) into a form from which the τ_ε dependence becomes more transparent by introducing the new function $\varphi(t) = \exp(t/\tau_\varepsilon)\,\mathrm{d}f/\mathrm{d}t$. Then from (7) and (9) we have

$$f(t) = f_\infty - \int_{t/T}^{\infty} \mathrm{d}x\, \varphi(x) \exp\left(-x\frac{T}{\tau_\varepsilon}\right) \tag{10}$$

with

$$\varphi(x) = \frac{1}{2\pi \mathrm{i}} \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \mathrm{d}\sigma \mathrm{e}^{\sigma x} \varphi(\sigma) \tag{11}$$

where the Laplace transformed function $\varphi(\sigma)$ is obtained from the transcendental equation

$$\varphi(\sigma) = \ln\left(\frac{\sigma}{\varphi(\sigma)} + \varepsilon^2\right) \tag{12}$$

which does not depend on τ_ε. In (10) $f_\infty = f(t=\infty) = f(\sigma = 0)$ is the dimensionless stationary current, which is obtained from

$$f_\infty = \ln\left(\frac{T}{\tau_\varepsilon f_\infty} + \varepsilon^2\right). \tag{13}$$

Now we can perform the inverse Laplace transformation in (11). For this purpose it is useful to replace the integral over σ in (11) by an integration over $z = \mathrm{e}^{\varphi}$ according to the equation

$$\sigma = (z - \varepsilon^2) \ln z. \tag{14}$$

Integrating by parts and performing some obvious calculations one obtains

$$\varphi(x) = \frac{\mathrm{i}}{2\pi x} \int \mathrm{d}\varphi' \exp[x\varphi'(\mathrm{e}^{\varphi'} - \varepsilon^2)] = \frac{\mathrm{i}}{2\pi x} \int \frac{\mathrm{d}z}{z} \exp[x(z - \varepsilon^2)\ln z]. \tag{15}$$

Now we choose an appropriate integration contour in the complex z-plane. For this end polar coordinates $z = z(\alpha) = \rho(\alpha)\exp(\mathrm{i}\alpha)$ are introduced and an integration over α along $\rho(\alpha)$ is performed:

$$\varphi(x) = -\frac{1}{2\pi T} \int \mathrm{d}\alpha \left(1 - \mathrm{i}\frac{\mathrm{d}\ln\rho(\alpha)}{\mathrm{d}\alpha}\right) \exp[x(\rho(\alpha)\mathrm{e}^{\mathrm{i}\alpha} - \varepsilon^2)(\ln\rho(\alpha) + \mathrm{i}\alpha)]. \tag{16}$$

The contour $\rho(\alpha)$ is chosen in such a way that the imaginary part of the exponent in (16) vanishes. This gives the following transcendental equation for $\rho(\alpha)$:

$$\rho \ln\rho \sin\alpha + \alpha(\rho\cos\alpha - \varepsilon^2) = 0. \tag{17}$$

If the integration is taken along this contour only the real part of the integral remains and we have

$$\varphi(x) = -\frac{1}{2\pi T} \int_{-\alpha_0}^{\alpha_0} \mathrm{d}\alpha \exp(x[(\rho(\alpha)\cos\alpha - \varepsilon^2)\ln\rho(\alpha) - \alpha\sin\alpha\rho(\alpha)]). \tag{18}$$

The contour $\rho(\alpha)$ is shown in figure 1 for three different values of the electric field ($\varepsilon = 0$, 1 and 2). In the large-scale representation of the polar coordinate plot (figure 1(b)) all three curves coalesce. It is seen from figure 1 that the integration over the deformed contour does

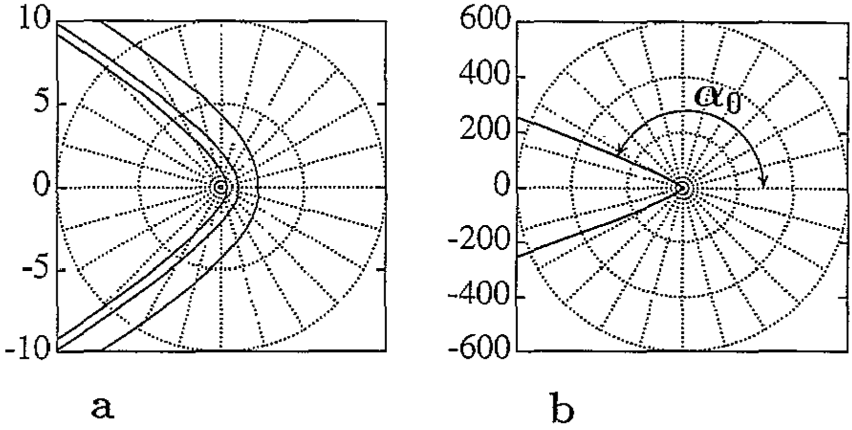

Figure 1. (a) Polar coordinate plot of the integration contours in (16) for $\varepsilon = 0$, 1 and 2 (from left to right). In the large-scale representation (b) the difference between the contours is not resolved. The angle α_0 of the asymptotic integration contour is shown in (b).

not change the value of the integral and that the desired Laplace transformed function is obtained.

At long times after the electric field is switched on ($t \gg T$, where $x \gg 1$) the main part of the integral in (18) comes from $\alpha \approx 0$. Expanding the integral around $\alpha = 0$ up to the second order we obtain

$$\varphi(x) = -[2\pi\rho_0(2 + \ln\rho_0)]^{-1/2} x^{-3/2} \exp(-x\rho_0 \ln^2 \rho_0) \tag{19}$$

where $\rho_0 = \rho(\alpha = 0)$ is the solution of

$$\rho_0 \ln(e\rho_0) = \varepsilon^2. \tag{20}$$

Now from (10) the asymptotic time dependence ($t \gg T$) is easily calculated:

$$f(t) = f_\infty + (\pi\rho_0(1 + \tfrac{1}{2}\ln\rho_0))^{-1/2} \left[\frac{\exp(-t/\tau_E)}{(t/T)^{1/2}} - \left(\frac{\pi T}{\tau_E}\right)^{1/2} \left(1 - \Phi\left(\sqrt{t/\tau_E}\right)\right) \right] \tag{21}$$

where $\Phi(x)$ is the error function and τ_E the following characteristic relaxation time:

$$\frac{1}{\tau_E} = \frac{\rho_0 \ln^2 \rho_0}{T} + \frac{1}{\tau_\varepsilon}. \tag{22}$$

The asymptotic representation (21) further simplifies if $t \gg \tau_E$:

$$f(t) = f_\infty + [2\pi(2 + \ln\rho_0)]^{-1/2} \frac{\tau_E \sqrt{T}}{t^{3/2}} e^{-t/\tau_E}. \tag{23}$$

Numerical results obtained from (10) and (18) and from (21) are shown in figures 2 and 3 by solid and dashed lines, respectively. In figure 2 the transient current is plotted for different values of the electric field ($\varepsilon = 0.9$, 1 and 1.1) and for the case where the inelastic scattering time τ_ε is much larger than T ($T = 10^{-5}\tau_\varepsilon$). At the delocalization field strength ($\varepsilon = 1$) a very slow current relaxation according to $f(t) \approx (\pi t/T)^{-1/2}$ is observed. If inelastic scattering becomes essential the slow current decay is replaced by an exponential relaxation as shown in figure 3 (for $T = 10^{-1}\tau_\varepsilon$, note the change of the scales). The long-time tail of the 2D current relaxation at the delocalization field strength occurs only at very large inelastic scattering times.

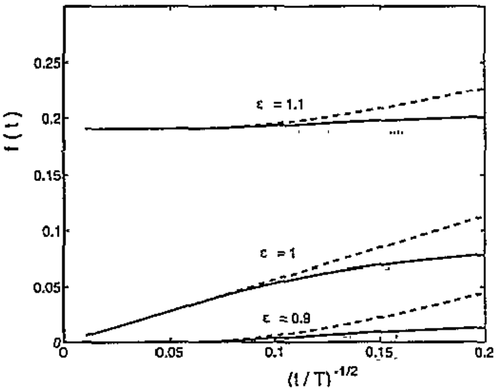

Figure 2. Time dependence of the normalized 2D current $f(t)$ for $T = 10^{-5}\tau_\varepsilon$ and $\varepsilon = E/E_0 = 0.9$, 1 and 1.1 (solid lines). The dashed lines are calculated from (21).

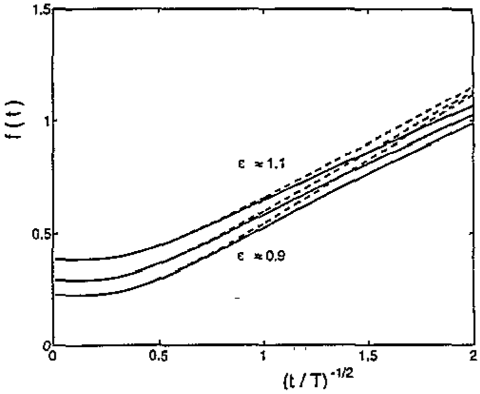

Figure 3. Time dependence of the normalized 2D current $f(t)$ for $T = 10^{-1}\tau_\varepsilon$ and $\varepsilon = E/E_0 = 0.9$, 1 and 1.1 (solid lines). The dashed lines are calculated from (21).

4. Current relaxation in one-dimensional systems

The current relaxation in a one-dimensional system ($d = 1$) can be treated in the same manner as the case $d = 2$ in section 3. In contrast to a two-dimensional electron gas now neither the edge field strength given by [8, 9]

$$E_0 = \frac{2\varepsilon_F}{e\pi\hbar N_F D_0} \tag{24}$$

nor the characteristic time

$$T = (2\pi\hbar N_F)^2 D_0 \tag{25}$$

depend exponentially on the disorder parameter. The self-consistent equation for the dimensionless current $f = D/D_0$ has the following form:

$$(1-f)^2\left[\frac{\sigma + T/\tau_\varepsilon}{f} + \varepsilon^2\right] = 1 \tag{26}$$

which together with (7), (10) and (11) allows the calculation of the time dependent current. For $d = 1$ (15) has now the following form:

$$\varphi(x) = \frac{1}{2\pi \mathrm{i} x}\int \mathrm{d}\varphi' \exp\left[x(1+\varphi')\left(\frac{1}{\varphi'^2} - \varepsilon^2\right)\right] \tag{27}$$

which is again evaluated by introducing polar coordinates $\varphi' = \rho(\alpha)\exp(\mathrm{i}\alpha)$ and by choosing the integration contour in such a way that the imaginary part of the integral (27) vanishes. This gives the condition

$$\cos\alpha = -\frac{\rho}{2}(1 + \varepsilon^2\rho^2) \tag{28}$$

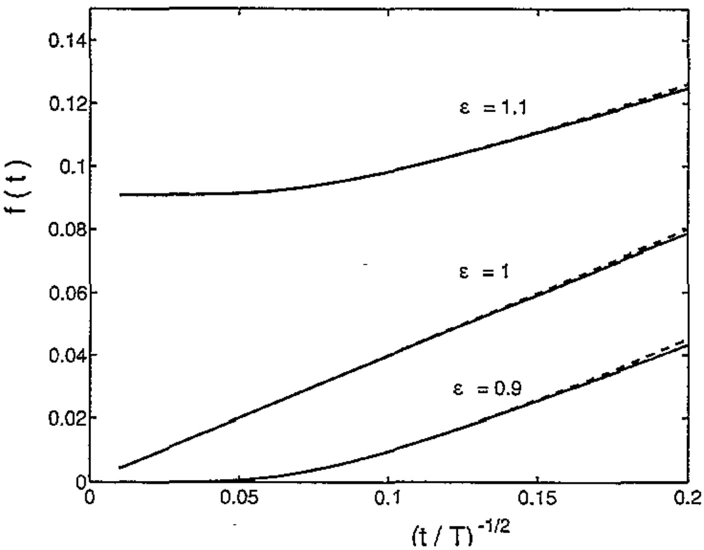

Figure 4. Time dependence of the normalized 1D current $f(t)$ for $T = 10^{-5}\tau_\varepsilon$ and $\varepsilon = E/E_0 = 0.9$, 1 and 1.1 (solid lines). The dashed lines are calculated from (31).

from which the following equation for $\varphi(x)$ results:

$$\varphi(x) = -\frac{1}{\pi x}\int_0^{\rho_0} d\rho \frac{1-(\rho^2/2)(1+2\varepsilon^4\rho^4+3\varepsilon^2\rho^2)}{\sqrt{1-((\rho/2)(1+\varepsilon^2\rho^2))^2}} \times \exp\left[x\left(\varepsilon^2\rho^2(1+\varepsilon^2\rho^2) - \frac{1}{\rho^2} - \varepsilon^2\right)\right]. \tag{29}$$

ρ_0 is the real solution of the algebraic equation

$$\varepsilon^2\rho_0^3 + \rho_0 - 2 = 0 \tag{30}$$

with $\rho_0 \leqslant 2$. (10), (11) and (29) are used in our numerical calculation.

As in section 2 an asymptotic expression ($t \gg T$) for the current decay can be derived from (10) and (29):

$$f(t) = f_\infty + \frac{\rho_0^2}{\sqrt{\pi(3-\rho_0)}}\left[\frac{e^{-t/\tau_E}}{\sqrt{t/T}} - \sqrt{\frac{\pi T}{\tau_E}}\left(1-\Phi\left(\sqrt{t/\tau_E}\right)\right)\right] \tag{31}$$

where the characteristic time τ_E is now given by

$$\frac{1}{\tau_E} = \frac{1}{\tau_\varepsilon} + \frac{2(1-\rho_0)^2}{T\rho_0^3} \tag{32}$$

and $f_\infty = f(\sigma = 0)$.

Numerical results obtained from (10) and (29) (solid lines) and from (31) (dashed lines) are shown in figure 4 for $T/\tau_\varepsilon = 10^{-5}$. These curves are very similar to the two-dimensional spectra in figure 2. Just at the delocalization field strength ($\varepsilon = 1$) the current decays according to the power law $f(t) = (2\pi t/T)^{-1/2}$.

5. Discussion

Because the current relaxation of one- and two-dimensional disordered systems, derived in sections 2 and 3, does not differ qualitatively, we mainly discuss the results obtained for $d = 2$. From (23) one obtains an exponential time decay of the current according to $j(t) \approx t^{-3/2} \exp(-t/\tau_E)$ with the characteristic relaxation time τ_E (22), which depends on the electric field via ρ_0 in (22). If inelastic scattering is neglected ($\tau_\varepsilon \to \infty$) τ_E diverges at the delocalization edge ($\varepsilon = 1$), as can be seen from (20) and (22). Consequently, the *current decay considerably diminishes if the electric field E approaches the delocalization* field strength E_0. In the near neighbourhood of the edge ($|\varepsilon - 1| \ll 1$) the time dependence of the current has the following form:

$$f(t) = \sqrt{\frac{T}{\pi t}} \exp\left[-\frac{t}{T}\left(\frac{E - E_0}{E_0}\right)^2\right] \tag{33}$$

from which at $E = E_0$ a power law dependence $f(t) \approx (\pi t/T)^{-1/2}$ results. This long-time *tail of the current relaxation is due to the existence of the delocalization edge.* The current relaxation dynamics accelerates if the electric field deviates from E_0.

A slow current decay at the edge field strength E_0 is also obtained for the one-dimensional case, where for $t/T \gg 1$ the asymptotic expression $f(t) = (2\pi t/T)^{-1/2}$ holds. If the electric field slightly deviates from E_0 (i.e., $|\varepsilon - 1| \ll 1$) the asymptotic time dependence has the form

$$f(t) = \sqrt{\frac{T}{2\pi t}} \exp\left[-\frac{t}{2T}\left(\frac{E - E_0}{E_0}\right)^2\right] \tag{34}$$

which differs from (33) only by the occurrence of an additional factor of two.

We conclude that the slowing down of the current relaxation at the delocalization edge is a characteristic property of weakly localized one- and two-dimensional systems. Experimentally this effect can be investigated by considering the destruction of the quantum interference due to a static electric field. For two-dimensional systems in particular the experiment is complicated by inelastic scattering (cf figure 3) because the current relaxation weakens remarkably only if $T \ll \tau_\varepsilon$ (cf (21) and (22)). Furthermore, according to (8) T increases exponentially with $\pi k_F \lambda/2$ and, therefore, very low carrier concentrations and extremely low lattice temperatures are required in order to measure the effect. This is seen from figure 5 where the dependences of the characteristic decay time T and the delocalization field strength E_0 on the electron concentration N and elastic scattering time $\tau = \lambda/v_F$ are shown. (In order to reproduce the results of the weak-localization theory we chose for the upper momentum cut-off $\kappa = \sqrt{2}/\lambda$ [9].) To satisfy the condition $T \ll \tau_\varepsilon$ the elastic scattering time τ should be small (figure 5(a)). On the other hand, with decreasing τ one observes a rapid increase of the edge field strength (figure 5(b)), which is accompanied by electron heating not considered here. Therefore, it seems to be not simple to find an optimal parameter set for the experiment. The situation is not so complicated for a one-dimensional system where the edge field strength is higher and the requirement $T \ll \tau_\varepsilon$ is not restrictive because $T = 4\tau$ (cf (25)) does not exponentially increase with the disorder parameter.

Measurements of the transient currents have been used extensively in the study of localized band tail states in amorphous semiconductors. It is a characteristic feature of non-crystalline systems that at different time scales different relaxation mechanisms are relevant.

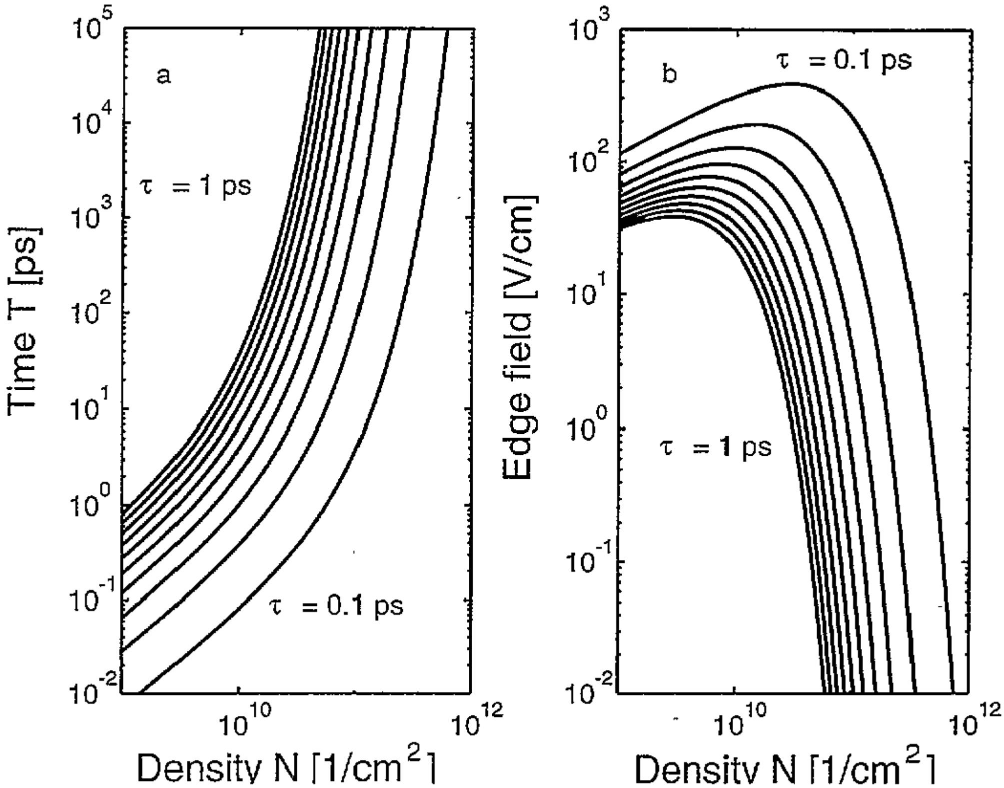

Figure 5. The relaxation time T (a) and delocalization field strength E_0 (b) as a function of the electron density. The elastic scattering times are $\tau = 0.1, 0.2, \ldots, 1$ ps and we used for the effective mass $m^* = 0.067m_0$ of GaAs.

At the short-time interval immediately after the electric field is applied the current relaxation is due to percolation over finite conducting clusters of the microscopically inhomogeneous system. After a long time period (i.e., at low frequencies) another relaxation mechanism becomes important, namely the current transport over some optimal percolation paths across the whole sample. These different relaxation processes at different time intervals give rise to a dispersive transient current (cf, e.g., [10, 11]), which was mainly investigated for three-dimensional systems on the basis of the hyperbolic decay functions [12]. The transient current of photoexcited carriers in amorphous thin films was treated in [13]. However, to our knowledge, there is no experimental evidence for the predicted long-time tail of the current relaxation, which is due to the existence of a delocalization field strength in one- and two-dimensional systems.

6. Summary

In this paper we considered the current relaxation of disordered one- and two-dimensional systems by considering the direct influence of a static electric field on the Cooper propagator and by neglecting hot-electron effects. Using the Einstein relation for the mobility the time dependent current is obtained from the inverse Laplace transformation of the diffusion coefficient. At the delocalization field strength both the one- and two-dimensional systems exhibit a power law current decay according to $j(t) \approx t^{-1/2}$ if inelastic scattering processes are neglected. This long-time current relaxation can easily be observed in disordered one-dimensional systems because the condition $T \ll \tau_\varepsilon$ for the characteristic decay time $T = 4\tau$ is not so restrictive. The situation alters completely for the two-dimensional case where both the delocalization field strength E_0 and the decay time T depend exponentially on the disorder parameter $k_F\lambda$. Therefore, the measurement of the predicted 2D long-time relaxation requires very low carrier densities and extremely low lattice temperatures.

Further theoretical and experimental research is desired in this direction.

The situation for a experimental verification might be better for an anisotropic two-dimensional electron gas where one expects that with increasing anisotropy the restrictive conditions of the two-dimensional case diminish.

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References

[1] Altshuler B L, Aronov A G and Khmelnitzky D E 1981 *Solid State Commun.* **39** 619
[2] Lei X L and Cai J 1990 *Phys. Rev.* B **42** 1574
[3] Hershfield S and Ambegaoker V 1986 *Phys. Rev.* B **34** 2147
[4] Kirkpatrick T R 1986 *Phys. Rev.* B **3** 780
[5] Vollhardt D and Wölfle P 1980 *Phys. Rev.* B **22** 4666
[6] Tsuzuki T 1981 *Physica* B **107** 679
[7] Delyon F, Simon B and Souillard B 1984 *Phys. Rev. Lett.* **52** 2187
[8] Prigodin V N 1980 *Zh. Eksp. Teor. Fiz.* **79** 2338; (Engl. transl. 1980 *Sov. Phys.–JETP* **52** 1185)
[9] Bryksin V V, Schlegel H and Kleinert P 1994 *Phys. Rev.* B **49** 13697
[10] Böttger H and Bryksin V V 1985 *Hopping Conduction in Solids* (Berlin: Akademie)
[11] Dyre J C 1991 *J. Non-Cryst. Solids* **135** 219
[12] Murayama K 1992 *Phil. Mag.* B **65** 749
[13] Zykov N V 1988 *Fiz. Tekh. Poluprov.* **22** 2095